MULTI-LINEAR FORMS, GRAPHS, AND L_p-IMPROVING MEASURES IN $\mathbb{F}_q^d$

Abstract

Given a connected ordered graph G on n vertices, consider the multi-linear form

$$\Lambda_G(f_1, f_2, ..., f_n) = \int_{x_1,..,x_n \in X} \prod_{(i,j) \in E(G)} K(x_i, x_j) \prod_{l=1}^{n} f_l(x_l) d\mu(x_l)$$

where $E(G)$ is the edge set for G. $f_i : X \to \mathbb{R}$ any functions, K and μ are suitable kernel and measure respectively.
Define $\Lambda_G(p_1, ..., p_n)$ as the smallest constant C > 0 such that the inequality

$$\Lambda_G(f_1, ..., f_n) \leq C \prod_{i=1}^{n} ||f_i||_{p_i}$$

holds for all non-negative valued functions f_i.
For our purposes, we will consider $K(x_i, x_j)$ to be the indicator function of a sphere evaluated at $x_i - x_j$. I will talk about how the structure of the graphs influences the exponents in the aforementioned inequality. Especially the sharpness of said results. This is joint work with A. Iosevich, D. Koh, T. Pham