ON A RESTRICTION PROBLEM OF HICKMAN AND WRIGHT FOR THE PARABOLA IN $\mathbb{Z}/N\mathbb{Z}$ FOR SQUAREFREE N

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Hickman and Wright proved an L^2 restriction estimate for the parabola Σ in $\mathbb{Z}/N\mathbb{Z}$ of the form

$$\left(\frac{1}{|\Sigma|}\sum_{m\in\Sigma}|\widehat{f}(m)|^2\right)^{\frac{1}{2}} \leq C_\epsilon N^\epsilon \cdot N^{-1}\left(\sum_{x\in(\mathbb{Z}/N\mathbb{Z})^2}|f(x)|^{\frac{6}{5}}\right)^{\frac{5}{6}}$$

for all functions $f : (\mathbb{Z}/N\mathbb{Z})^2 \to \mathbb{C}$ and any $\epsilon > 0$, and that this bound is sharp when N has a large square factor, and especially for $N = p^2$ for p a prime. In contrast, Mockenhaupt and Tao proved in the special case $N = p$ the stronger estimate

$$\left(\frac{1}{|\Sigma|}\sum_{m\in\Sigma}|\widehat{f}(m)|^2\right)^{\frac{1}{2}} \leq CN^{-1}\left(\sum_{x\in(\mathbb{Z}/N\mathbb{Z})^2}|f(x)|^{\frac{4}{3}}\right)^{\frac{3}{4}}.$$

We extend the Mockenhaupt-Tao bound to the case of squarefree N, proving

$$\left(\frac{1}{|\Sigma|}\sum_{m\in\Sigma}|\widehat{f}(m)|^2\right)^{\frac{1}{2}} \leq C_\epsilon N^\epsilon \cdot N^{-1}\left(\sum_{x\in(\mathbb{Z}/N\mathbb{Z})^2}|f(x)|^{\frac{4}{3}}\right)^{\frac{3}{4}},$$

and discuss applications of this result to uncertainty principles and signal recovery.

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