The Fourier ratio, complexity and time series analysis

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Abstract

The main purpose of this talk is to investigate the degree to which the ratio

$$\mathrm{FR}(f) = \frac{\|\widehat{f}\|_1}{\|\widehat{f}\|_2}$$

can indicate the complexity or learnability of the signal $f : \mathbb{Z}_N \to \mathbb{C}$. Using results from Bourgain and Talagrand, it can be shown that this ratio is large when f is concentrated in a random set, and moreover that in the case that $\mathrm{FR}(f)$ is small, f is well-approximated by a low-degree polynomial. These ideas can be further exploited to connect $\mathrm{FR}(f)$ to Kolmogorov complexity, VC-dimension, statistical query dimension, and more.