THE FOURIER RATIO AND CHANG'S LEMMA

ABSTRACT. Given a function $f : \mathbb{Z}_N \to \mathbb{C}$, we denote by $\widehat{f}$ its Fourier transform which is given by

$$\widehat{f}(m) = \frac{1}{\sqrt{N}} \sum_{x \in \mathbb{Z}_N} e^{-2\pi i x m/N} f(x).$$

We introduce the function $FR(f) = \frac{\|\widehat{f}\|_1}{\|\widehat{f}\|_2}$, and examine how this ratio tells us the extent to which we can approximate f by a trigonometric polynomial. Finally, we prove a generalized version of Chang's lemma and show that the sparse spectrum approximation of f has some additive structure.